

Conditions for supersonic bent Marshak waves

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Supersonic radiation diffusion approximation is a useful way to study the radiation transportation. Considering the bent Marshak wave theory in 2-dimensions, and an invariable source temperature, we get the supersonic radiation diffusion conditions which are about the Mach number $M > 8(1 + \sqrt{\varepsilon})/3$, and the optical depth $\tau > 1$. A large Mach number requires a high temperature, while a large optical depth requires a low temperature. Only when the source temperature is in a proper region these conditions can be satisfied. Assuming the material opacity and the specific internal energy depend on the temperature and the density as a form of power law, for a given density, these conditions correspond to a region about source temperature and the length of the sample. This supersonic diffusion region involves both lower and upper limit of source temperature, while that in 1-dimension only gives a lower limit. Taking SiO_2 and the Au for example, we show the supersonic region numerically.

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I. INTRODUCTION

Radiation transport in plasma is complicated because of complicated course between photon and material. The local material density and temperature affect the emission and absorption of the radiation, while the non-local radiation affects the radiation energy flux. When the material is optically thin, the radiation flux can bleach through and ionize the material, but when the material is optically thick, the photons of the external source are absorbed and re-emitted many times before reaching the heating front. When the opacity is so large that the mean free path of photon is much less than the distance over which temperature are changing, the radiation diffusion approximation can be used. Assuming the radiation is isotropic, and considering the gray diffusion approximation, the radiation diffusion act as heat waves just like the thermal conduction. Radiation heat waves as a specifical form of radiation diffusion are very common and important in the plasma physics, and in particular in inertial confinement fusion(ICF) where we need to know how the laser energy is absorbed by the hohlraum inner wall. It also appears in the astrophysical situations such as supernova explosions and the penetration of radiation into the interstellar medium.

Generally speaking, the radiation diffusion can be described as the interaction between radiation and material. Most of the time, researchers tend to study the supersonic radiation diffusion in which the time scales of radiant energy exchange are quite smaller than hydrodynamic time scales, so that the penetration of radiation and energy deposition can occur while the density of material does not change yet. That's to say, we can treat the density of material as a constant in supersonic radiation diffusion. This treatment in supersonic radiation diffusion condition brings significant convenience to both the theory and the experiment.

The earliest solution of the supersonic heat waves was worked out in 1958, by Marshak[1] who demonstrated that the location of radiation front is proportional to \sqrt{t} , and after the front, the radiant flux is taken up by the material and converted into internal energy. Following the work of Marshak, many more accurate solutions are worked out[2–5, 9]. However, these solutions are entirely confined into one-dimensions which is difficult to describe the bent shape of the heat front. In 2000, using Ω -laser, Back et al designed an experiment to study the supersonic radiation diffusion in (50mg/cc) SiO_2 and (40mg/cc) Ta_2O_5 and found the curvature of the radiation front[10]. To explain such a curvature, Hurricane and Hammer developed an analytic solution(Bent Marshak wave) in two-dimensions[11]. This theory indicates that energy loss to the sleeve which surrounds the material causes the curvature of the radiation front.

Experimentally, in addition to the Back's experiments, Afshar-ra and Hoarty et al studied the supersonic heat wave in low density foam composed of CH compound and a little other elements used for determining temperature [12–17], and Bozier did the similar work in the high Z gas composed of Xe[18]. Generally, when designing the

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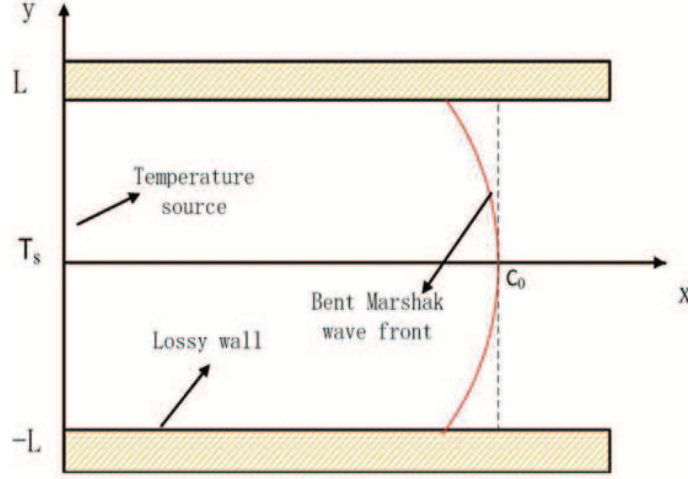


FIG. 1: The picture that bent heat waves propagate in a pipe with a constant temperature source on the left and a lossy wall bounds the problem at $y = L$.

supersonic radiation diffusion experiment, one must consider two conditions. first, the speed of the radiation front must be larger than the sound speed of the material which requires a high temperature and a low density of the material. Second, to use the radiation diffusion approximation, the material must be optically thick that requires a low temperature and a high density. The two conditions have a opposite dependence on the temperature and density of material. Therefore, the temperature and the density must satisfy a specific condition so that the supersonic radiation diffusion approximation can be used. On the other hand, when this radiation heat wave propagates in the material, the temperature drops, so does the wave's speed. So, how long the supersonic radiation heat wave can propagate is a problem which need to be worked out.

In one-dimension, the condition for the supersonic radiation has been shown in [19]. In this paper, considering a power-law opacity, we discuss the condition for the supersonic bent marshak wave in two-dimensions, and study how long the supersonic Marshak wave can propagate. This paper is organized as follows. In Sec. I, we introduce the analytic solution of bent Marshak wave. In Sec. II, we generally discuss the condition for supersonic Marshak wave. In section III, we study the condition for supersonic Marshak wave theoretically by using SiO_2 and Au for example. In sec IV, we give the numerical result. At last, we give the conclusions in Sec. IV.

II. BASIC THEORY OF BENT MARSHAK WAVE

A. The expansion of the solution and boundary conditions

The propagating of heat waves can be described as the interaction between radiation and the material. In this course, the material gets internal energy from the radiation flux, which can be expressed as[6]

$$\rho \frac{\partial e}{\partial t} = \frac{4}{3} \nabla \left[\frac{1}{\rho \kappa_R} \nabla (\sigma T^4) \right], \quad (1)$$

where ρ is the material density, e is the material specific internal energy, T is the material and radiation temperature when considering the local thermodynamic equilibrium, $\sigma = 1.03 \times 10^{12} \text{ergs}/(\text{cm}^2 \text{seV}^4)$ is the Stefan-Boltzman constant, and κ_R is the Rosseland mean opacity.

In the supersonic case, considering the density ρ and Rosseland mean opacity as a constant, and away from the radiation front, the general solution of $T^4(x, t)$ in two-dimensions pipe as FIG. 1 can be given as

$$T^4 = \sum_{n=0}^{\infty} \cos(k_n y) [A_n(t) e^{k_n x} + B_n(t) e^{-k_n x}], \quad (2)$$

where k_n is decided by the boundary conditions, A_n and B_n are time-dependent coefficients respectively. According to the form of temperature, the form of the radiation front can be chosen to be

$$x_f(x, t) = \sum_{n=0}^{\infty} c_n(t) \cos(k_n y), \quad (3)$$

which describes a bent radiation front. Hurricane and Hammer think that the bent shape of the radiation front is caused by a non-ideal boundary at the walls, $y = \pm L$, which has a albedo, a . Not all the energy flux into the boundary can be reflected as a form of re-emission, so, the boundary condition at walls is defined as[11]

$$F|_{y=\pm L} = -\frac{4}{3\rho\kappa_R} \nabla(\sigma T^4) = -(a-1)\sigma T^4, \quad (4)$$

while the boundary condition at two sides are $T|_{x=0} = T_s$ and $T|_{x=x_f} = 0$ at respectively. According to the boundary conditions at walls and the general solution of temperature, ones get the eigenvalue condition

$$\tan(k_n L) = \frac{\varepsilon}{k_n L}, \quad (5)$$

which gives the eigenvalues

$$\begin{aligned} k_0 &= \frac{\sqrt{\varepsilon}}{L}, \\ k_n &= \frac{n\pi}{L} + \frac{\varepsilon}{Ln\pi}, n \geq 1, \end{aligned} \quad (6)$$

where $\varepsilon = \frac{3}{4}\rho\kappa_R L(1-a)$ is a small value, considering the albedo of walls tends to be 1. Hurricane and Hammer expand the solution, Eq. (2) in $\sqrt{\varepsilon}$ and give[11]

$$\begin{aligned} \frac{T^4}{T_s^4} &= -(1 + \frac{\varepsilon}{3}) \cos(\sqrt{\varepsilon} \frac{y}{L}) \frac{\sinh[k_0(x - c_0)]}{\sinh(k_0 c_0)} \\ &+ \frac{4\varepsilon}{\pi^2} \sum_{n=1}^{\infty} \cos(k_n y) \frac{(-1)^{n+1}}{n^2} \frac{\sinh[k_n(x - c_0)]}{\sinh(k_n c_0)} + \mathcal{O}(\varepsilon^{3/2}), \end{aligned} \quad (7)$$

where c_0 is zeroth coefficient of the radiation front, and we can approximately consider it as the position of the front for leading order.

B. Bent radiation front

According to the expression of temperature, combining with the equation of motion(EOM) of the radiation front[5]

$$\rho e \dot{x}_f = -\frac{4\sigma}{3\kappa_R \rho} \nabla T^4 \Big|_{x_f}, \quad (8)$$

found by Hammer and Rosen, where e is the specific internal energy of material at T_s , the coefficients of the radiation wave front can be restricted as[11]

$$\begin{aligned} \dot{c}_0 &= \frac{4\sigma T_s^4}{3\rho^2 \kappa_R e} (1 + \frac{\varepsilon}{3}) \frac{\sqrt{\varepsilon}}{L \sinh(\sqrt{\varepsilon} c_0 / L)} + \mathcal{O}(\varepsilon^2), \\ \dot{c}_n &= \frac{8\sigma T_s^4}{3\rho^2 \kappa_R e} \frac{(-1)^n \varepsilon}{n^2 \pi^2} \frac{k_n}{\sinh(k_n c_0)} + \mathcal{O}(\varepsilon^2), n \geq 1. \end{aligned} \quad (9)$$

Considering the leading order, thinking $c_0 \simeq x_f$ and integrating Eq. (9), the expression of the front is given as

$$x_f(y, t) \simeq \frac{L}{\sqrt{\varepsilon}} \cosh^{-1} \left[\frac{D\varepsilon t}{2L^2} + 1 \right] \cos(\sqrt{\varepsilon}y/L), \quad (10)$$

where the parameter $D = (1 + \varepsilon/3) \frac{8\sigma T_s^4}{3\rho^2 \kappa_R e}$ which represents the velocity of the radiation front is just like the diffusion constant of the Marshak wave[1]. The parameter ε decided by the albedo, determines the shape of the radiation front. According to Eq. (10), if the wall absorb more energy and re-emission less energy, the parameter ε will be larger, and the curvature of the radiation front will also be larger.

III. CONDITIONS FOR RADIATION-DOMINATED AND SUPERSONIC

Radiative energy density is small compared to the material energy density, therefore it often does not dominate the energy balance. However, if the speed of radiation front is much greater than the sound speed, the radiative energy flux dominates the material energy flux and therefore can determines energy flow. In some way, we think the conditions for Radiant energy flux domination and the supersonic are equal approximately. Generally, the ratio of these two term,

$$\gamma = \frac{\text{radiative energy flux}}{\text{material energy flux}} = \frac{\sigma T^4}{\rho e C_S}, \quad (11)$$

must be greater than 1. Setting $y = 0$, we expand the form of the radiation front and get

$$x_f|_{y=0} \approx \sqrt{Dt} - \frac{L}{12} \sqrt{\frac{2}{\varepsilon}} \left(\frac{D\varepsilon t}{2L^2} \right)^{3/2} + \mathcal{O}(\varepsilon^{3/2}). \quad (12)$$

Using Eq. (12), considering the velocity of radiation front u is equal to x_f/t approximately, and defining the optical depth $\tau \approx \rho \kappa_R x_f = x_f/l_0$ (l_0 represents the mean free path of the photon), we get

$$\sigma T^4 = u \tau \rho e \frac{3}{8(1 + \varepsilon/3)}. \quad (13)$$

Substituting Eq. (13) into Eq. (11), we get

$$\gamma = \frac{3}{8(1 + \varepsilon/3)} \cdot M \cdot \tau, \quad (14)$$

where The Mach number, M , is the ratio of the radiative front velocity to the heated material sound speed and its value increases as the importance of the radiative flux increases.

According to the definition, the radiative energy flux dominating requires

$$M \cdot \tau > \frac{8(1 + \varepsilon/3)}{3}, \quad (15)$$

which is similar to the condition gotten by Back[10]. To satisfy the condition of diffusion approximation, we need $\tau = x_f/l_0 > 1$ which means the represents the mean free path of the photon is much less than the distance over which temperature is changing. On the other hand, we hope the hydrodynamic density perturbations at the radiation front remain as small as possible. Since the changes of material density have some relationships with the velocity of the wave front, that is $\frac{\rho_2}{\rho_1} \approx (1 + M^{-2})$ [23]. The Mach number is required to be large enough that ones may ignore the density perturbation. Therefore, the conditions for radiation-dominated are

$$M > \frac{8(1 + \varepsilon/3)}{3}, \quad \tau > 1, \quad (16)$$

on which the hydrodynamic density perturbations will be less than 10%.

We assume the opacity and the specific internal energy depend on temperature and the density as a form of power law[5, 20–22], which can be expressed as

$$\begin{aligned}\kappa_R &= \tilde{\kappa}_R \left(\frac{T_s}{T_0} \right)^{-\alpha} \left(\frac{\rho}{\rho_0} \right)^\lambda, \\ e &= e_0 \left(\frac{T_s}{T_0} \right)^\beta \left(\frac{\rho}{\rho_0} \right)^{-\mu},\end{aligned}\tag{17}$$

where $\alpha, \lambda, \beta, \mu$ are the positive constant, T_0 and ρ_0 represent the specific temperature and density respectively. At the same time, we can give the expression of sound speed as

$$\begin{aligned}C_s &= \sqrt{\frac{\partial P_i}{\partial \rho} + \frac{\partial P_e}{\partial \rho}} = \sqrt{(1+Z) \frac{k_B T}{Am_p}} \\ &= \Gamma \left(\frac{T_s}{T_0} \right)^{1/2},\end{aligned}\tag{18}$$

where k_B and m_p are Boltzmann constant and the mass of proton respectively, Γ is a constant with a dimension m/s . Therefore the conditions of the radiation-dominated and supersonic can be rewritten as

$$\begin{aligned}M = \frac{\dot{x}_f}{C_s} &= \frac{4\sigma T_0^4}{3\tilde{\kappa}_R \rho_0^2 e_0 \Gamma L} \left(\frac{T_s}{T_0} \right)^{4+\alpha-\beta} \left(\frac{\rho}{\rho_0} \right)^{\mu-2-\lambda} \\ (1+\varepsilon/3) \frac{\sqrt{\varepsilon}}{\sinh(\sqrt{\varepsilon} x_f/L)} &> \frac{8(1+\varepsilon/3)}{3}, \\ \tau = \rho \kappa_R x_f &= \rho_0 \tilde{\kappa}_R x_f \left(\frac{T_s}{T_0} \right)^{-\alpha} \left(\frac{\rho}{\rho_0} \right)^{1+\lambda} > 1,\end{aligned}\tag{19}$$

from which, we know a higher source temperature and the lower density lead to a longer distance that the supersonic heat wave can propagate noticing that the Mach number is proportional to $\sqrt{\varepsilon}/\sinh(\sqrt{\varepsilon} x_f/L)$.

IV. NUMERICAL RESULT FOR TWO KINDS OF MATERIAL

In the supersonic diffusion, the material density can be considered as a constant, and the optical depth is proportional to x_f . When radiation wave front reach a critical position x_1 , the optical depth become large enough that the diffusion approximation can be used. While the radiation heat wave is propagating in the material, the temperature becomes lower, and the velocity of the radiation front becomes smaller. When radiation front reach another critical position x_2 , the velocity of the radiation wave front become subsonic. Therefore, when giving a certain density, every source temperature corresponds to a critical position. usually, x_1 is a little smaller than x_2 . That to say, for a given temperature, when the front position is in the region $x_1 < x < x_2$, we can use the supersonic radiation diffusion approximation. We use two kinds of material, SiO_2 and Au for example to illustrate this region.

A. supersonic diffusion region for SiO_2

Taking SiO_2 for example, considering it as ideal gas, and assuming the SiO_2 plasma with a density $\rho_0 = 50 \text{ mg/cc}$ is fully ionized at the characteristic temperature $T_0 = 10^6 \text{ K}$, according to Eq. (18), sound speed can be given as

$$C_s(T) = 6.74 \times 10^4 [\text{m/s}] \left(\frac{T}{T_0} \right)^{1/2}.\tag{20}$$

The specific internal energy of SiO_2 can be expressed as

$$e = \frac{k_B T}{(\gamma-1)Am_p} = 6800 [\text{MJ/kg}] \left(\frac{T}{T_0} \right).\tag{21}$$

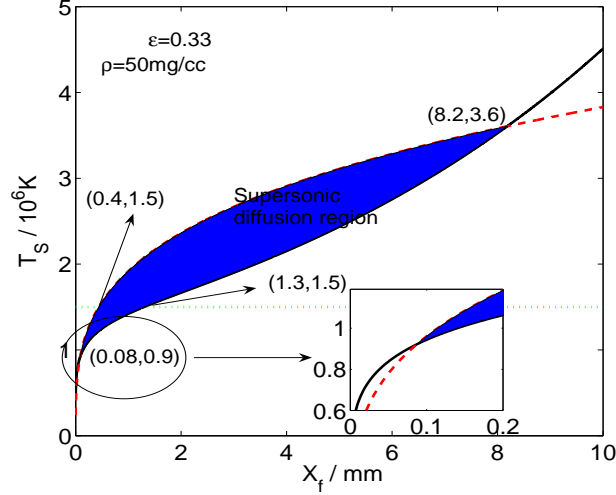


FIG. 2: Curves for Mach number condition and the optical depth condition with the SiO_2 density 50mg/cc and the parameter $\varepsilon = 0.29$, the solid line corresponds to the supersonic condition, the dash line corresponds to the optical depth condition and the dotted line represents temperature $T = 1.5 \times 10^6 k$.

By fitting the numerical result of opacity of SiO_2 , the paper [19] give the relationship

$$\kappa_R = 175 m^2/kg \left(\frac{T}{T_0} \right)^{-3.3} \left(\frac{\rho}{\rho_0} \right)^{0.64} \quad (22)$$

Therefore, the conditions for supersonic diffusion in SiO_2 are given as

$$\begin{aligned} 0.474 \left(\frac{T_s}{10^6 k} \right)^{5.8} \left(\frac{\rho}{50 \text{mg/cc}} \right)^{-2.64} \frac{\sqrt{\varepsilon}}{\sinh(\sqrt{\varepsilon} x_f / L)} &> \frac{8}{3}, \\ 8.75 \frac{x_f}{\text{mm}} \left(\frac{T_s}{10^6 k} \right)^{-3.3} \left(\frac{\rho}{50 \text{mg/cc}} \right)^{1.64} &> 1 \end{aligned} \quad (23)$$

If we fix the density to 50mg/cc , and change the sign $' > '$ in Eq. (23) to $' = '$, every source temperature T_s corresponds to a front position x_1 according to the first condition, and according to the second condition, corresponds to another front position x_2 . These temperature and front position are plotted in FIG. 2. the solid line corresponds to the supersonic condition, the dash line corresponds to the optical depth condition and the dotted line represents temperature $T = 1.5 \times 10^6 k$. The condition $M > \frac{8}{3}(1 + \varepsilon/3)$ corresponds to the the region above the solid line, and the condition $\tau > 1$ corresponds to the region below the the dash line. Noticing the FIG. 2, when setting the parameter $\varepsilon = 0.33$ which indicates there is 38% of the radiation energy flux absorbed by the wall, the dash line and the solid line cross each other two times, at the point $(0.08, 0.93)$ and $(8.2, 3.6)$ respectively. As long as the point (T_s, x_F) locates in the blue region shown in the picture, the supersonic radiation diffusion approximation can be used. When the temperature is as low as 0.9eV , the supersonic region comes into being. When the temperature becomes higher and tends to be 2eV , the distance supersonic heat wave propagate tends to be longest. When the temperature is around 3.6eV , we can use supersonic radiation diffusion no longer. When setting the temperature to be $1.5 \times 10^6 k$, the supersonic radiation diffusion region is $0.44\text{mm} < x < 1.3\text{mm}$. Therefore, it is reasonable to study the supersonic radiation diffusion with the source temperature $\sim 1.5\text{eV}$ and the length of sample $0.5\text{mm} - 1.25\text{mm}$.

In the same way, when fixing the density to 40mg/cc and also set ε to be 0.33 , we get a similar supersonic region, seeing FIG. 3(left). However the total region has a shift in the left-down direction. When setting the temperature to be $1.5 \times 10^6 k$, the supersonic radiation diffusion region is $0.6\text{mm} < x < 2.0\text{mm}$.

When fix the density to 40mg/cc and also set ε to be 0.8 , we get a narrower supersonic region comparing to the case of $\varepsilon = 0.33$, seeing FIG. 3(right). The upper limit of temperature becomes smaller while the lower limit keep the same with that in case of $\varepsilon = 0.33$. It indicates that a bigger ε which means more energy is absorbed by walls makes the supersonic region narrower.

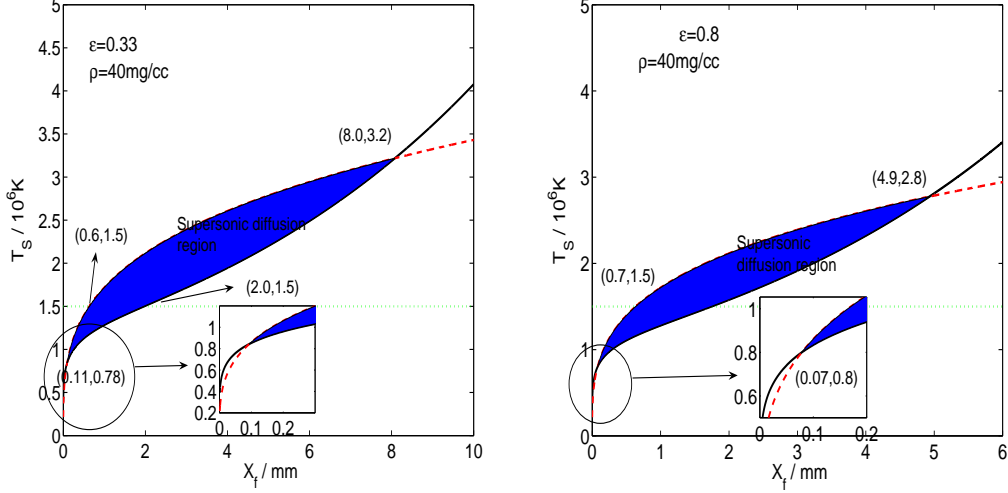


FIG. 3: Curves for Mach number condition and the optical depth condition with the SiO_2 density 40mg/cc , with the parameter $\varepsilon = 0.33$ (left) and $\varepsilon = 0.8$ (right), the solid line corresponds to the supersonic condition, the dash line corresponds to the optical depth condition and the dotted line represents temperature $T = 1.5 \times 10^6 k$.

B. supersonic diffusion region for Au

Taking Au for example, and knowing the average degree of ionization is about 50 at the temperature 1.9heV [24], the sound speed can be given as

$$Cs(T) \simeq 3.36 \times 10^4 [m/s] \left(\frac{T}{10^6 k} \right)^{1/2}. \quad (24)$$

According to the result of Hammer[5], the opacity and the specific internal energy can be described as

$$\begin{aligned} \kappa_R &= 395.5 m^2/kg \left(\frac{T}{10^6 k} \right)^{-1.5} \left(\frac{\rho}{50 mg/cc} \right)^{0.2} \\ e &= 2235 [MJ/kg] \left(\frac{T}{10^6 k} \right)^{1.6} \left(\frac{\rho}{50 mg/cc} \right)^{-0.14}. \end{aligned} \quad (25)$$

In the same way, substituting these expressions into the supersonic conditions, we get

$$\begin{aligned} 1.28 \left(\frac{T_s}{10^6 k} \right)^{3.9} \left(\frac{\rho}{50 mg/cc} \right)^{-2.06} \frac{\sqrt{\varepsilon}}{\sinh(\sqrt{\varepsilon} x_f / L)} &> \frac{8}{3}, \\ 19.75 \frac{x_f}{mm} \left(\frac{T_s}{10^6 k} \right)^{-1.5} \left(\frac{\rho}{50 mg/cc} \right)^{1.2} &> 1 \end{aligned} \quad (26)$$

which means Au has a higher opacity with the same temperature and density comparing to SiO_2 . According to these two conditions, setting the parameter $\varepsilon = 0.33$, and $\rho = 50\text{mg/cc}$ we give the supersonic diffusion region, seeing FIG. 4. These two lines also cross two times, while the supersonic region is much larger than SiO_2 thanks to the larger opacity. Noticing the picture, we can see, when the temperature is as low as 0.4heV , the supersonic region comes into being. When the temperature becomes higher and tends to be 10heV , the distance supersonic heat wave propagate tends to be longest. When the temperature is around 37heV , we can use supersonic radiation diffusion no longer.

V. CONCLUSIONS AND DISCUSSION

In the supersonic radiation diffusion theory, the radiant energy flux dominates the material energy flux that gives a limit about the Mach number and the optical depth. By using the 1-dimensions theory, Back et al give this limit as

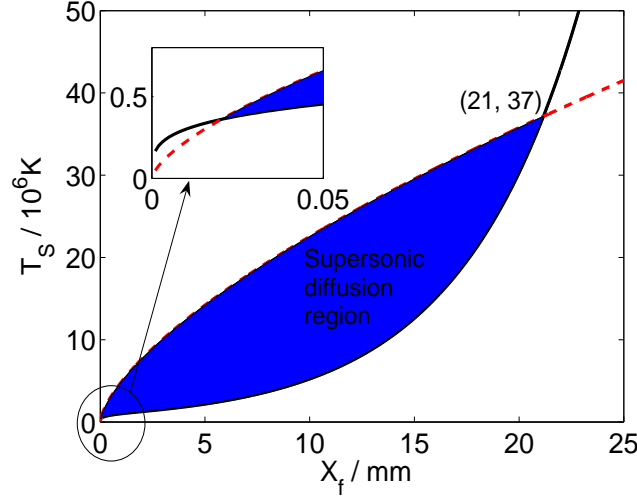


FIG. 4: Curves for Mach number condition and the optical depth condition with the Au density 50mg/cc and parameter $\varepsilon = 0.33$.

$M\tau > 3$ [10] and point out that the density perturbation is less than 10% when Mach number $M > 3$. The conditions for supersonic and the optical depth corresponds to a region about the source temperature T_s and the location of wave front x_f . Comparing to Back's result, considering the bent Marshak wave theory in two dimensions, we get the supersonic radiation diffusion conditions which are about the Mach number $M > 8(1 + \sqrt{\varepsilon})/3$, and the optical depth $\tau > 1$. It indicates the Mach number depend on how much energy are lost at the wall $y = \pm L$. Obviously, when more radiation energy are lost at the wall, a larger Mach number is required to ensure the radiation energy flux dominates. It is worth mentioning that, for a given density, the theory in 2-dimension gives both lower and upper limit about source temperature and the length of the sample, which makes sure the supersonic heat wave exists, while the theory in 1-dimension only gives a lower limit. This means the theory in 2-dimension makes the conditions for supersonic diffusion wave stricter comparing to that in 1-dimension.

Acknowledgments

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